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RE-3613

M. A. / M. Sc. (Part - II) Examination

April / May – 2010

Mathematics : Paper - 5021

(Computational Fluid Dynamics)

Time : 3 Hours] [Total Marks : 42

Instrucitons :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **M. A. / M. Sc. - 2**

Name of the Subject :

☛ **5021 - MATHEMATICS**

☛ Subject Code No. : **3 6 1 3** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) Each question carry equal marks.
(3) Attempt all questions.
(4) All questions are compulsory.

Q1(a) Derive the Continuity equation for the model of an infinitesimally small fluid element fixed in space.

(b) Explain the physical meaning of $div\ \vec{V}$

(c) Derive the second order central difference formula with respect to x.

OR

Q1(a) Derive the Navier Stoke's equation in conservative form where the infinitesimally small fluid element moving with the flow.

(b) Define the following terms:
(i) Stress (ii) Shear stress (iii) Normal stress

Q2(a) Derive the Energy equation in conservative form for the model of an infinitesimally small fluid element moving with the flow.

OR

Q2(a) Classify the Heat equation, Wave equation and Laplace equation using crammer's rule.

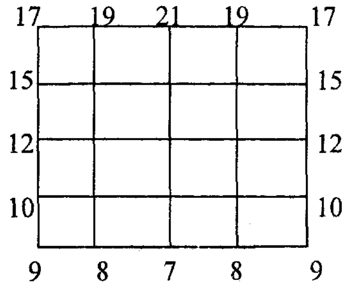
(b) Solve the heat equation along with the boundary and initial conditions as follows:
$u(0,0) = 0; u(h,0) = 0.5; u(2h,0) = 0.5;\ u(3h,0) = 0;\ u(0,t) = u(1,t) = 0.$
Using Richardson and Du-fort-Frankel Method for $h = \frac{1}{3}\ and\ k = \frac{1}{36}$.

Q3 Discuss Stability of the following finite difference Schemes:
(i) Schmidt Method.
(ii) Richardson Method.
(iii)Crank Nicolson Method.

OR

Q3(a) Derive Richardson and Laasonen Finite Difference Method to solve the Heat conduction equation $u_t = u_{xx}$

(b) Solve the Partial differential equation $\nabla^2 u = 0$ for the figure given below where h=k carrying out two iterations.



Q4(a) Explain, how the system of quasi-linear Partial differential equations are classified using eigen values. Classify: $(1 - M_\infty^2)u_x' + v_y' = 0$

$u_y' - v_x' = 0$; where M_∞ is the free stream Mach number

Q4(b) Find the solution of initial value problem $u_{tt} = u_{xx}$; $0 \le x \le 1$ subject to the initial conditions: $u(x,0) = \sin^3 \pi x$; $0 \le x \le 1$

$u_t(x,0) = 0$; $0 \le x \le 1$

The boundary condition $u(0,t) = u(1,t) = 0$; $t > 0$ by using Leapfrog Method assume $h = \frac{1}{4}$ and $k = \frac{1}{5}$. Solve upto two time steps.

OR

Q4(a) Find the solution of initial value problem $u_t = u_{xx}$ subject to the initial condition: $u(x,0) = \cos\left(\frac{\pi x}{2}\right)$; $-1 \le x \le 1$ and the boundary conditions

$u(-1,t) = u(1,t) = 0$; $t > 0$

by using Crank Nicolson Finite Difference Scheme. Assume $h = \frac{1}{3}$ and $k = \frac{1}{4}$. Solve upto two time steps.

(b) Check the stability by Fourier series method of first order hyperbolic equation discretized by explicit Euler forward time and forward space difference method.

Q5(a) Find the approximating solution of $\left[\frac{d^2u}{dx^2} - u\right] + x^2 = 0, \quad 0 < x < 1$, $u(0) = 1, \quad x\frac{du}{dx}\Big|_{x=1} = 0.$ using Galerkin method

(b) Explain Finite Element Method to solve Poisson's equation for a Triangular element.

OR

Q5(a) Use Finite Element Method to solve the Boundary value problem $\nabla^2 u = -2; 0 \le x, y \le 1$ with the condition u=0 on the boundary of the square $0 \le x \le 1; 0 \le y \le 1$

(b) Explain the basic steps to solve any physical problem using Finite element Method.